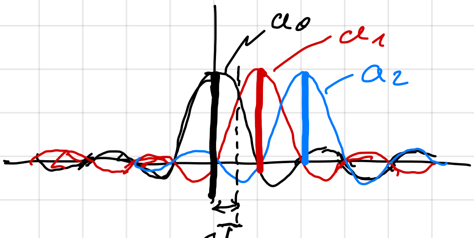
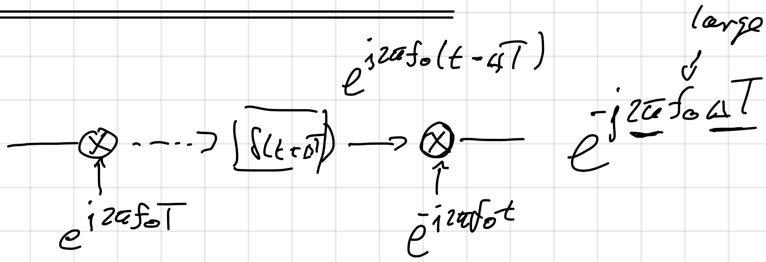
Wireless Receivers

10/9/20

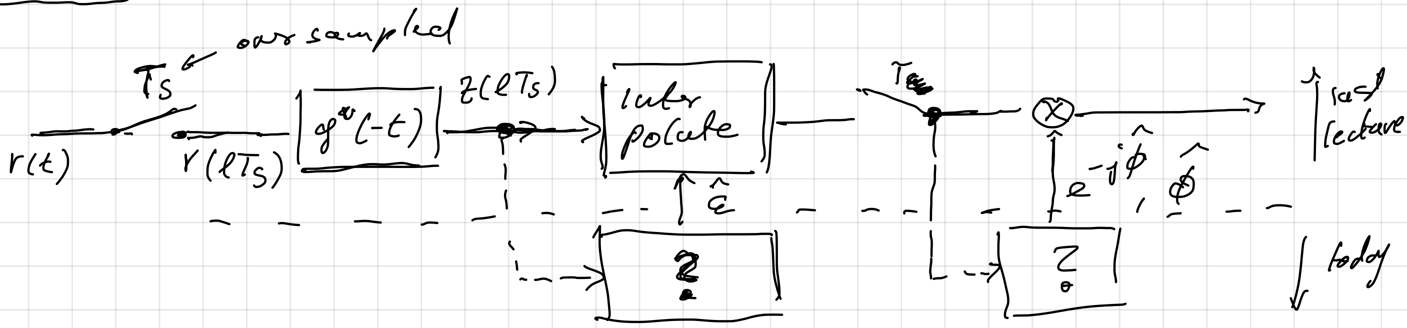
Parameter Estimation





$0 \le \varepsilon \le 1$ Sampling-time offset εT_s

Receiver:



How to get to $\hat{\varepsilon}$ and $\hat{\phi}$?

Receiver Parameter Estimation

$\boxed{\text{Formulate the ML condition}}$

we have: $\underline{r} = [\dots, r(-2T_s), r(-T_s), r(0), r(T_s), r(2T_s), \dots]$ (oversampled samples)

Unknowns: transmitted symbols $\underline{a} = [\dots, a[-2], a[-1], a[0], a[1], a[2], \dots]$

Time offset: ε

Phase offset: ϕ

$$\{\hat{\underline{a}}, \hat{\varepsilon}, \hat{\phi}\} = \arg\max_{\underline{a}, \varepsilon, \phi} \Pr(\underline{r} \mid \underline{a}, \varepsilon, \phi)$$

Blind estimation: data is unknown

a) joint estimation: search over all options

b) Estimate parameters one-by-one
→ marginalize over the others = take the average over the others

- Timing $\hat{\varepsilon}$: $\Pr(\underline{r} \mid \varepsilon) = \int_{-\pi}^{+\pi} \sum_{\underline{a}} \underbrace{\Pr(\underline{a})}_{\text{uniform}} \Pr(\underline{r} \mid \varepsilon, \underline{a}, \phi) \underbrace{p(\phi)}_{\text{uniform } -\pi \dots +\pi} \, d\phi$

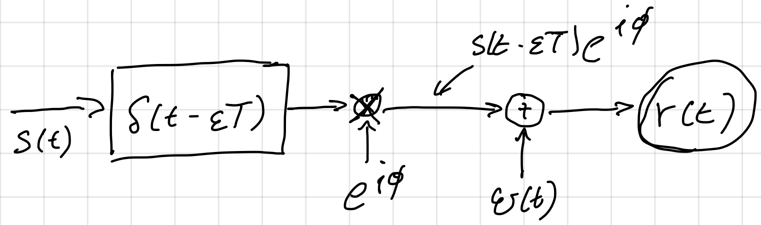
- Phase $\Pr(\underline{r}|\phi) = \int \sum_{\underline{a}} \Pr(\underline{a}) \Pr(\underline{r}|\phi, \underbrace{\varepsilon, \underline{a}}) \, p(\varepsilon) \, d\varepsilon$

3) Estimation one after another
↳ for each parameter assume we know the ones we estimated before *perfectly*

- Timing ε: $\hat{\varepsilon} = \arg\max_{\varepsilon} \Pr(\underline{r}|\varepsilon)$
- Phase ϕ: $\hat{\phi} = \arg\max_{\phi} \Pr(\underline{r}|\varepsilon = \hat{\varepsilon}, \phi)$
- Data $\underline{a}$: $\hat{a} = \arg\max_{\underline{a}} \Pr(\underline{r}|\varepsilon = \hat{\varepsilon}, \phi = \hat{\phi}, \underline{a})$

ML criterion for AWGN channel

$\rightarrow$ Goal is to define $Pr(\underline{r} | \ldots)$



$w(t)$: $\mathcal{CN}(0,\sigma^2) \sim e^{-\frac{|w(t)|^2}{2\sigma^2}}$

$$\underline{r(t)}: \mathcal{CN}\Big(\underbrace{s(t-\varepsilon T)e^{j\phi}}_{\text{Tx Sequence incl. phase offset}}, \sigma^2\Big) \sim e^{-\frac{|r(t)-s(t-\varepsilon T)e^{j\phi}|^2}{2\sigma^2}}$$

Ref RX signal (no noise) + given ε, ϕ
$s(t-\varepsilon T)e^{j\phi}$

Insted of comparing to the delayed ref sequence $s(t-\varepsilon T)e^{j\phi}$ we compare $s(t)e^{j\phi}$ to „un-delayed" RX sequence/samples $r(t+\varepsilon T)$ and write $\underline{r(t+\varepsilon T)}$ as a function of $\underline{r(t)}$ $[\underline{r(\ell T_s)}]$

Consider $P_r(\underline{r}|r^{ref})$

For one sample: $p(r|r^{ref}) = \exp(-|r-r^{ref}|^2)$

$$\sum_n |r_n - r_n^{ref}|^2$$

for multiple samples: $p(\underline{r}|\underline{r}^{ref}) = \prod_n p(r_n|r_n^{ref}) = \exp(-\|\underline{r}-\underline{r}^{ref}\|^2)$

$$= \exp\left(-\cancel{\|\underline{r}\|^2} + 2\mathrm{Re}\{(\underline{r}^{ref})^H \underline{r}\} - \|\underline{r}^{ref}\|^2\right)$$

Note: $|\underline{r}|^2$ is indep. of the parameters we try
has no impact on finding the maximizing parameters

$$L(\underline{r}|r^{ref}) \sim \exp\left(2\mathrm{Re}\{(\underline{r}^{ref})^H \underline{r}\} - \|r^{ref}\|^2\right)_{r(t+\varepsilon T)}$$

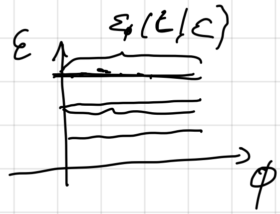
Compare the „corrected" sampled sequence $r(t+\varepsilon T)$ with sampled Tx output (w/o time delay)

$$r(t) \rightarrow \underbrace{r(\ell T+\varepsilon T)}_{= z_\ell(\varepsilon)} \quad \text{with} \quad r^{ref} \rightarrow s(t) = \underbrace{\sum_k a_k g(t-kT)e^{\phi j}\Big|_{t=\ell T}}_{a_\ell \cdot e^{j\phi}}$$

$$L(z_\ell(\varepsilon)|a_\ell e^{j\phi}) = \exp\left(+2\mathrm{Re}\{a_\ell^* e^{-j\phi} z_\ell\} - |a_\ell|^2\right)$$

$$L(\underline{z}|\underline{a}e^{j\phi}) = \exp\left(+2\mathrm{Re}\{\underline{a}^H e^{-j\phi}\underline{z}\} - \|a\|^2\right)$$

$$= \exp\Big(\sum_{l=-L}^{+L}\left[2\mathrm{Re}\{a_l^* z_l(\varepsilon)e^{-j\phi}\}\right] - \cancel{\underbrace{\sum_{l=-L}^{+L}|a_l|^2}_{\text{for large } L \to \text{const.}}}\Big)$$



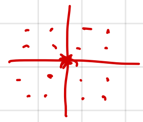
$$= \exp\Big(\sum_{l=-L}^{+L} 2\mathrm{Re}\{a_l^* z_l(\varepsilon)e^{-j\phi}\}\Big)$$

derived from our Rx samples @ $r(lT_s)$

Blind estimation: average over all possible $\underline{a}$, or a_l $l=-L\ldots L$

$\mathcal{E}_a\{L(\underline{z}\,|\,\underline{a}e^{j\phi})\} \to \underline{a}$ is in the exponent

$$\mathcal{E}_a\{f(\underline{a})\} \neq f(\mathcal{E}_a\{\underline{a}\}) \quad \times$$



Workaround: Taylor expansion

$$\exp\left(2\mathrm{Re}\{\textstyle\sum a_l^* z_l(\varepsilon)e^{-j\phi}\}\right) = \cancel{1} + \underbrace{2\mathrm{Re}\{\textstyle\sum a_l^* z_l(\varepsilon)e^{-j\phi}\}}_{1 \;\to 0} + \underbrace{\left(\mathrm{Re}\{\textstyle\sum a_l^* z_l(\varepsilon)e^{-j\phi}\}\right)^2}_{2}$$

1 $\mathcal{E}_{\underline{a},\phi}\{2\mathrm{Re}\{\sum a_l^* z_l(\varepsilon)e^{-j\phi}\}\} = \mathrm{Re}\{\underbrace{\mathcal{E}_{\underline{a},\phi}\{\sum a_l^* z_l(\varepsilon)e^{-j\phi}\}}_{\to 0}\}$

2 $= \left(\frac{1}{2}\sum_l a_l^* z_l(\varepsilon)e^{-j\phi} + \frac{1}{2}\sum a_l z_l^*(\varepsilon)e^{+j\phi}\right)^2$ ← remove the Re{..}

$$= \sum_l\left(|a_l|^2\,|z_l(\varepsilon)|^2 + \frac{1}{2}\mathrm{Re}\{a_l^2(z_l(\varepsilon))^2 e^{j2\phi}\}\right)$$

Average $\mathcal{E}_a$

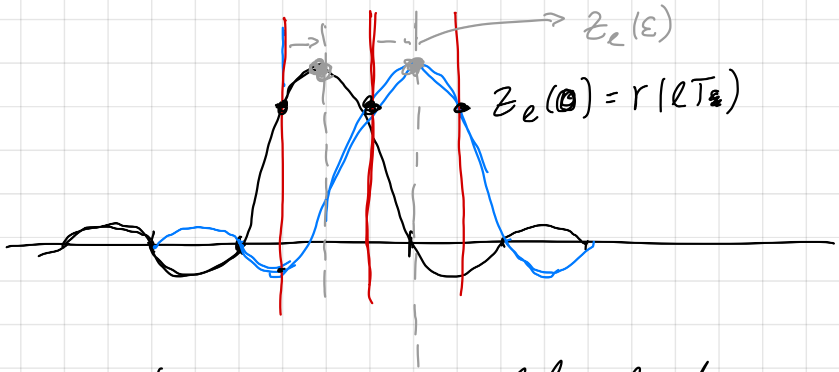
$$= \mathcal{E}_\phi\Big\{\underbrace{\sum_l \underbrace{\mathcal{E}_a\{|a_l|^2\}}_{\to\text{const.}}|z_l(\varepsilon)|^2}\; + \frac{1}{2}\mathrm{Re}\{\underbrace{\mathcal{E}_a\{a_l^2\}}_{\text{const.}}(z_l(\varepsilon))^2 e^{j2\phi}\}\Big\}$$

Average over $\phi \sim [0, 2\pi[$ → eliminates the 2nd term

$$L(\varepsilon) \sim \sum |z_l(\varepsilon)|^2 \qquad \text{with } z_l(\varepsilon) = r(lT+\varepsilon T)$$

write as a fct. of $r(lT_s)$

$$\hat{\varepsilon} = \arg\max_{\varepsilon} \sum |z_l(\varepsilon)|^2$$



$\underline{L(\varepsilon)} \sim \sum |z_\ell(\varepsilon)|^2$

with $z_\ell(\varepsilon) = \underline{r(\ell T_s + \varepsilon)}$ ✗

we only have $\underline{r(\ell T_s)}$

write $z_\ell(\varepsilon)$ as a fct. of the samples I have $\underline{r(\ell T_s)}$

How to construct $z_\ell(\varepsilon)$ from $r(\ell T_s)$

Interpolate: band-limited signal

1) write as Fourier Series, evaluate at $\ell T + \varepsilon$

Fourier coeffs found around the ℓth symbol

$$\underbrace{|z(\ell T + \varepsilon)|^2}_{z_\ell(\varepsilon)} = \sum_{n=-\infty}^{\infty} c_n^{(\ell)} \underbrace{e^{j 2\pi n \varepsilon}}_{\text{offset}}$$

2) Get $c_n^{(\ell)}$ from our samples $r(pT_s)$ around $r(\ell T)$

the symbol ℓ

$\ell T,\ \ell T + T_s,\ \ell T + 2T_s,\ \ell T + 3T_s \cdots \ell T + (M-1) T_s$

M-fold oversampling

$$c_n^{(\ell)} = \sum_{p=0}^{M-1} |r(\ell T + pT_s)|^2 e^{-j \frac{2\pi}{M} np}$$

3) Substitute into $L(\varepsilon) \sim \sum_\ell |z_\ell(\varepsilon)|^2 = \sum_{n=-\infty}^{+\infty} \underbrace{\sum_\ell \sum_{p=0}^{M-1}}_{\text{over all samples}} |r(\ell M T_s + pT_s)|^2 \cdot e^{-j \frac{2\pi}{M} np} e^{2\pi n \varepsilon}$

Note: $r(t)$ is band limited to $\pm \frac{1}{2T}$

$\Rightarrow |r(t)|^2$ is bandlimited too, but to $\frac{1}{T}$

real valued

$\rightarrow$ only c_{-1}, c_{+1}, and $c_0 \neq 0$

$$\underline{L(\varepsilon)} \sim c_0 + \underbrace{c_{-1} e^{-2\pi \varepsilon j} + c_{+1} e^{-2\pi \varepsilon j}}_{\text{compl. conj pairs}} = \underline{c_0} + \underbrace{2 \operatorname{Re}\{c_1 e^{-2\pi j \varepsilon}\}}_{\max}$$

should be real for max

$$c_1 = \sum_\ell \sum_p |r(\underbrace{\ell M T_s}_{\ell T} + pT_s)|^2 e^{-j \frac{2\pi}{M} p} = \text{FT of squared Rx samples}$$

$$\hat{\varepsilon} = \arg(c_1) \frac{1}{2\pi}$$

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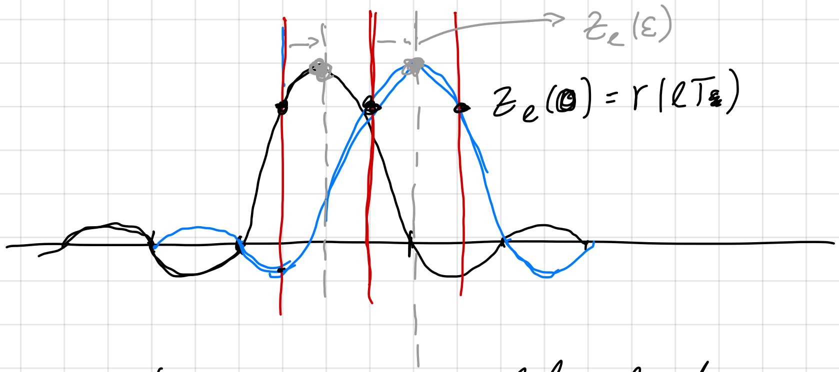

$L(\varepsilon) \sim \sum |z_\ell(\varepsilon)|^2$

with $z_\ell(\varepsilon) = r(\ell T_s + \varepsilon)$ ✗

we only have $r(\ell T_s)$

write $z_\ell(\varepsilon)$ as a fct. of the samples I have $r(\ell T_s)$

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1) write as Fourier Series, evaluate at $\ell T + \varepsilon$

Fourier coeffs found around the ℓth symbol

$$\underbrace{|z(\ell T + \varepsilon)|^2}_{z_\ell(\varepsilon)} = \sum_{n=-\infty}^{\infty} c_n \underbrace{e^{j2\pi n\varepsilon}}_{\text{offset}} \cdot e^{j2\pi n \frac{1}{MN}\ell M}$$

N T-spaced samples
M-fold oversampled

2) Get c_n from our samples $r(pT_s)$

$$c_n = \sum_{p=0}^{M-1} |r(pT_s)|^2 e^{-j\frac{2\pi}{MN}np}$$

3) Substitute into

$$z_\ell(\varepsilon) = \sum_n c_n e^{j2\pi n\varepsilon} e^{j2\pi \frac{n}{MN}\cdot \ell M}$$

$$\sum_\ell z_\ell(\varepsilon) = \sum_\ell \underbrace{\sum_n c_n e^{j2\pi n\varepsilon} e^{j2\pi \frac{n}{N}\ell}}_{\neq 0 \text{ except } n = \pm N,\ n=0}$$

$$\Rightarrow \sum_\ell z_\ell(\varepsilon) = \sum_\ell \left(c_{-N} e^{+j2\pi N\varepsilon} + c_0 + c_N e^{-j2\pi N\varepsilon}\right)$$

$$c_0 + \underbrace{c_{-N} e^{-2\pi\varepsilon j} + c_{+N} e^{-2\pi\varepsilon j}}_{\text{compl. conj. pairs}} = c_0 + \underbrace{2\,\mathrm{Re}\left\{c_N e^{-2\pi j\varepsilon}\right\}}_{\max}$$

should be real for max

$$c_N = \sum_p |r(pT_s)|^2 \underbrace{e^{-j\frac{2\pi}{M}p}}_{\text{for } M=4:\ 1;\,-j;\,-1;\,j}$$

$$\hat{\varepsilon} = \arg(c_N)\frac{1}{2\pi}$$

Phase Estimation → Assume that $\varepsilon = \hat{\varepsilon}$

Timing is already corrected

Data aided: know data $\underline{a}^{(0)}$ = known

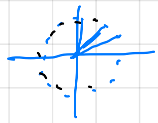
$$L(\phi) = L(\underline{z} \mid \underline{a} = \underline{a}^{(0)}, \varepsilon = \hat{\varepsilon}, \phi) = \exp\left(\Re\left\{\underbrace{\sum_{\ell} a^{(0)*} z_\ell(\hat{\varepsilon})\, e^{j\phi}}_{\to \text{Real valued}}\right\}\right) \to \max$$

$$\hat{\phi} = \arg\left(\sum_{\ell} a_\ell^{(0)*} z_\ell(\hat{\varepsilon})\right) \checkmark$$

Ambiguity of $\pm\pi$

Non-data aided: not knowing $\underline{a}$

PSK:



Problem: $L(\phi)$ depends on $\underline{a}$ or a_ℓ

Idea: Remove dependency on a_ℓ ← single symbol

- Assumption: PSK-modulation $z_\ell(\hat{\varepsilon}) = a_\ell e^{j\phi}$ $\quad a_\ell = e^{j2\pi \frac{1}{Q} \overset{\text{data}}{d_\ell}}$

Note: $(a_\ell)^Q = \left(e^{j\frac{2\pi}{Q} d_\ell}\right)^Q = \underbrace{e^{j2\pi d_\ell}}_{\equiv 1 \text{ regardless of } d_\ell}$ $\quad d_\ell$ = Int.

Consider modified rx-signal $\underline{[z_\ell(\hat{\varepsilon})]^Q} = \underbrace{a_\ell^Q}_{1} e^{jQ\phi}$

$$L(\phi) = \Re\left\{\sum_{\ell} \underbrace{(a_\ell^{(0)*})^Q}_{1} \underline{(z_\ell(\hat{\varepsilon}))^Q e^{-j\phi}}\right\}$$

$$\hat{\phi} = \underbrace{\arg\left(\sum (z_\ell(\hat{\varepsilon}))^Q\right)}_{\pm\pi} \frac{1}{Q}$$

Ambiguity of $\pm\frac{\pi}{Q}$